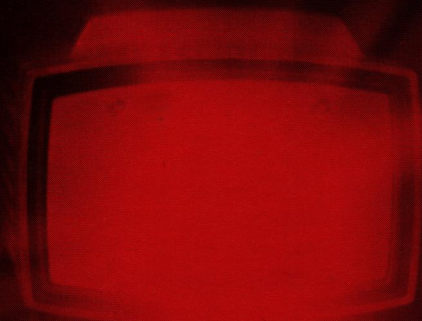


# MEASUREMENT OF STRAIGHTNESS OF TRAVEL

Laser Measurement System  
Application Note 156—5



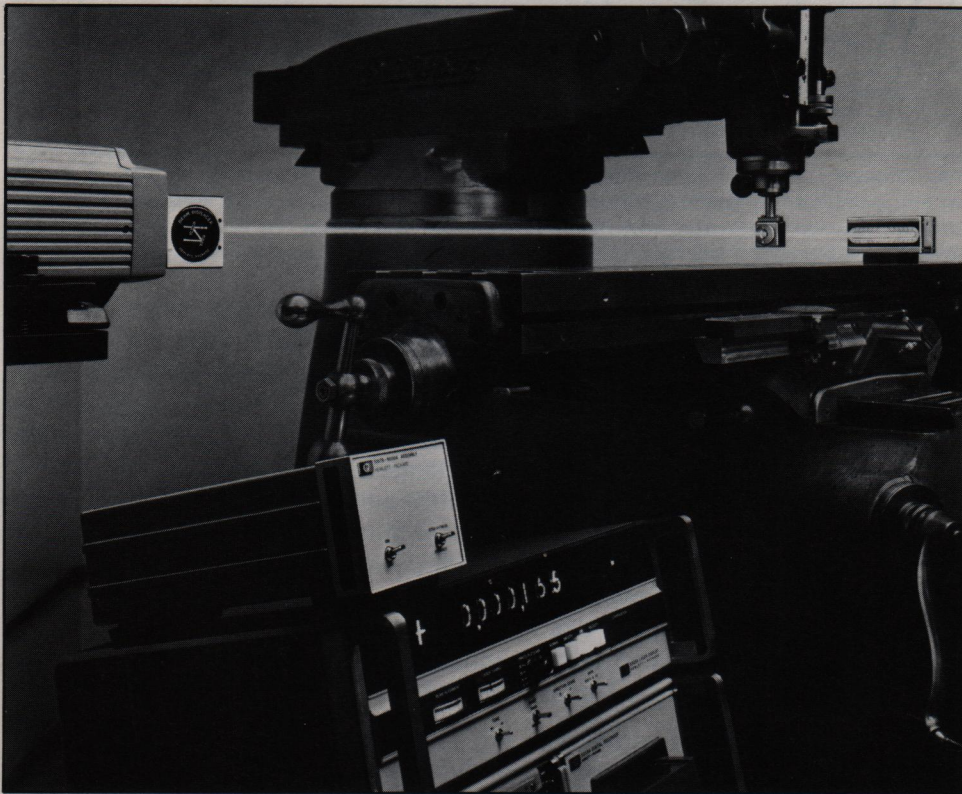


## Introduction

It is becoming increasingly important in the machine tool industry to determine the precise geometry as well as the positioning accuracy of machine slides. The problem is particularly acute for multi-axis machine tools, where the method most often used for single-axis machines, the Abbe Comparator Principle<sup>1</sup>, cannot be followed. Present methods allow accurate measurement of machine slide positioning, but can measure geometry only in terms of angular quantities such as pitch and yaw. The only reliable tools available for determining straightness of travel have been the short straightedge and the optical flat.\* These devices are adequate in cases where the total machine slide travel is two feet or less, but it is not feasible to produce physical standards to accommodate travel longer than this.

Traditionally, attempts have been made to overcome this problem by resorting to an optical or mechanical substitute such as a mobile fluid, a stretched wire, an alignment telescope, or an alignment laser. Unfortunately, such attempts have proved much less accurate than the short straightedge or optical flat, which have remained the most accurate straightness standards for the last century.

The straightness interferometer, as shown in Figure 1, provides a more direct answer to the straightness problem: use small optical flats to measure straightness over a long travel. Using this approach the straightness interferometer has



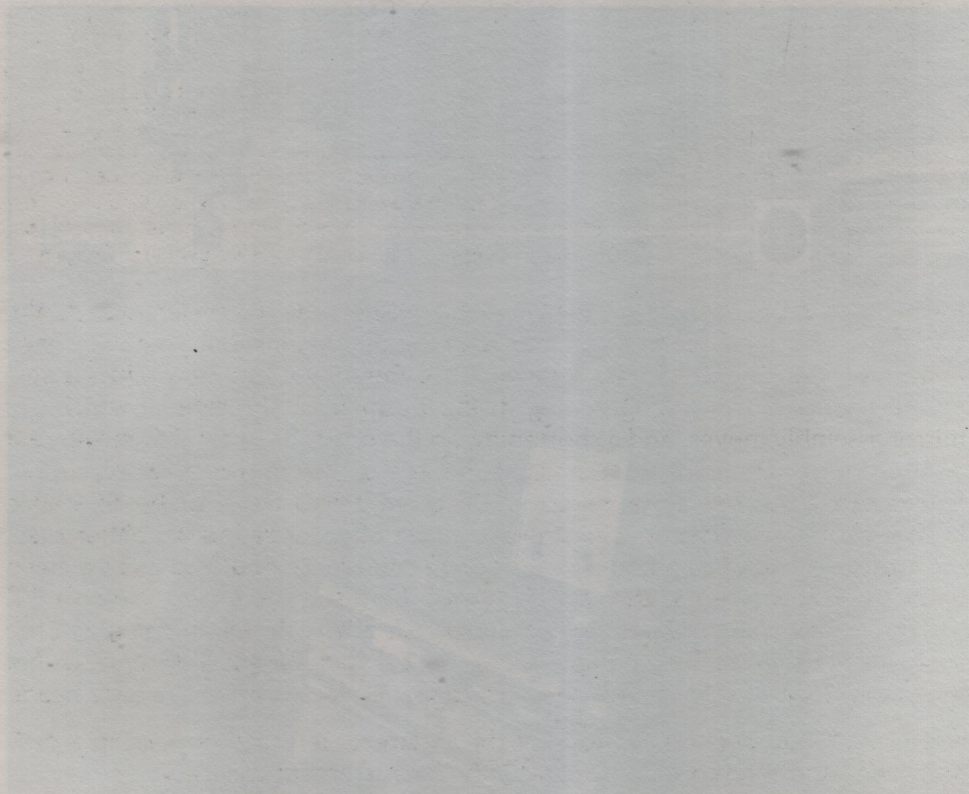
**Figure 1.** The straightness interferometer, an optical accessory to the 5526A Laser Interferometer System, measures straightness, parallelism, and perpendicularity of machine-tool and measuring machine coordinate motions over a range of approximately 100 feet or 30 metres. Basic accuracy is  $\pm$  five microinches per foot of travel ( $\pm$  .4 micrometres/metre).

\*A glass plate carefully lapped and polished for maximum flatness.



achieved an accuracy of five microinches per foot of travel for travel up to one hundred feet, without requiring state-of-the-art optical flats. The interferometer is presently available in two versions: a short-range instrument that has one-microinch resolution and measures straightness over ten feet of travel, and a long-range instrument that has ten-microinch resolution and measures straightness of travel over one hundred feet of travel. Both have an inherent accuracy of five microinches per foot.

Besides high accuracy, the straightness interferometer shares another advantage with the optical flat. The accuracy of both devices can be improved by a technique known as "reversing the straightedge" in which a second straightness measurement is taken with the reference straightedge reversed so that it faces in a direction opposite that of the first measurement. By averaging the two measurements, the error of the interferometer, or optical flat, can be reduced to the limit of repeatability of the device under test.



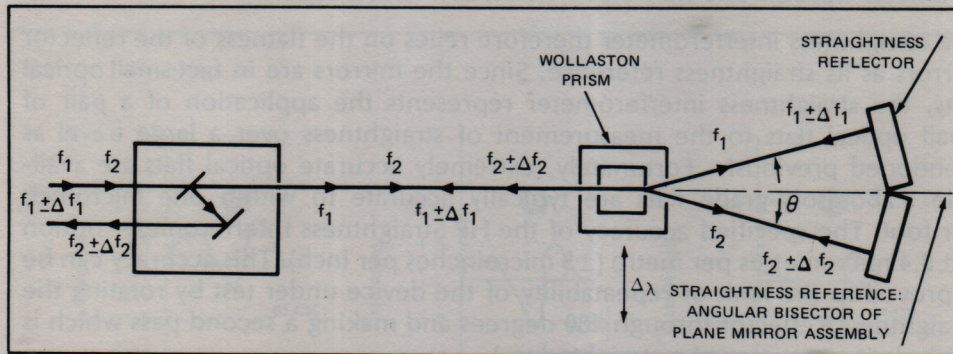


## Principle of Operation

The two-frequency beam exiting from the HP 5526A Laser Measurement System is transmitted through the Wollaston prism<sup>2</sup> interferometer. Because the composite refractive index of the prism is different for the two planes of polarization which distinguish  $f_1$  and  $f_2$ ,  $f_1$  and  $f_2$  exit with a small included angle (see Figure 2).

They are reflected back by two plane mirrors rigidly mounted at an included angle precisely matched to that of the Wollaston prism interferometer.  $f_1$  and  $f_2$  therefore recombine within the prism. The combined beam is returned coaxially with the exit beam to the partial mirror in the Straightness Adaptor. The majority of this returning signal is reflected down to a mirror which reflects the return beam back into the lower aperture of the Laser Head and thus to a demodulating polarizer and photodetector.

The interferometer measures relative lateral displacement between the interferometer and the reflecting mirror axis. Whether the measurement will be in a horizontal or vertical plane depends on the orientation of the mirrors and the prism within its mount.



**Figure 2.** The straightness interferometer causes the two differently polarized frequency components in the laser beam to diverge. After reflection from two precisely plane mirrors, the two components recombine and interfere in the interferometer. Any transverse motion of the interferometer relative to the mirror bisector causes an optical path change that is detected and displayed.

Relative lateral displacement between the prism and the mirrors affects the difference in optical path lengths between the two beams causing a difference in accumulated fringe counts. Movement of the mirror assembly with respect to the beams causes a lengthening in the beam from the side to which the mirror assembly moves, and a shortening on the opposite beam. For movement of the interferometer with respect to the axis of the mirror assembly, there is an optical path length change within the prism proportional to the difference in the refractive indices specific to each plane of polarization. In either case, for a relative lateral translation  $x$ , the fringe counts accumulated will be given by  $2x \sin \theta/2$ , where  $\theta$  is the included angle between the beams. However, if the beam moves with respect to the mirror axis any path length change in the air space is balanced by a compensating optical path length difference within the prism. Thus the device is insensitive to spatial deviations of the laser beam.

Small pitch, yaw, or roll motions of the interferometer do not create a path difference and therefore do not affect the measurement accuracy.

To give a correct readout on the display, the fringe counts must be multiplied by the reciprocal of  $2 \sin \theta/2$ , which for the value of  $\theta$  used is 36. An electronic Resolution Extender is included to take care of this.  $\theta$  is about  $1\frac{1}{2}$  degrees for the Short-range Interferometer, and one-tenth that for the Long-range version.



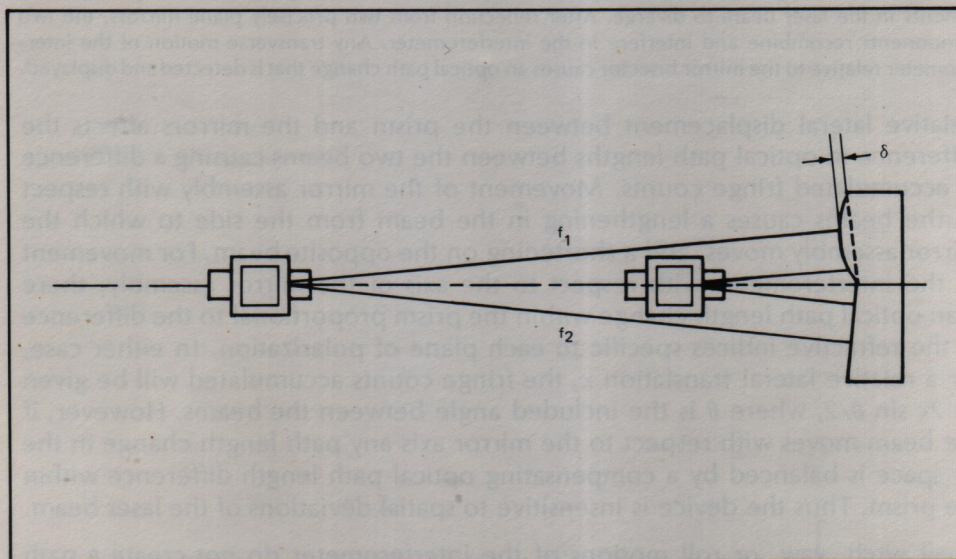
## Potential Sources of Error Associated With Using the HP Straightness Interferometer

The HP Straightness Interferometer, like all measurement devices, is subject to various types of errors through misapplication of the instrument or operation in poor environments. These types of errors can be broken down into the categories of systematic and random errors.

### Systematic Errors

The limiting accuracy of the HP Straightness Interferometer is directly related to the difference in flatness of the two plane mirrors that serve as the interferometer reflector. For example, suppose one of the mirrors is flat but the other is convex by a small amount,  $\delta$ , as shown in Figure 3. Even though the Wollaston prism, or straightness interferometer, were to travel in a straight path with respect to the mirror axis, the optical path length of  $f_1$  would be shortened by an amount equal to  $\delta$  with respect to the optical path of  $f_2$ , and this would be interpreted by the interferometer as "out of straightness" of the interferometer travel.

The straightness interferometer therefore relies on the flatness of the reflector mirrors as its straightness reference. Since the mirrors are in fact small optical flats, the straightness interferometer represents the application of a pair of small optical flats to the measurement of straightness over a large travel as mentioned previously. Fortunately, extremely accurate optical flats are available. Laboratory-grade flats are typically accurate to within one microinch per foot. The specified accuracy of the HP Straightness Interferometer option is  $\pm 0.4$  micrometres per metre ( $\pm 5$  microinches per inch). This accuracy can be improved to the limit of repeatability of the device under test by rotating the Straightness Reflector through 180 degrees and making a second pass which is equivalent to reverse of a straightedge.<sup>3</sup>

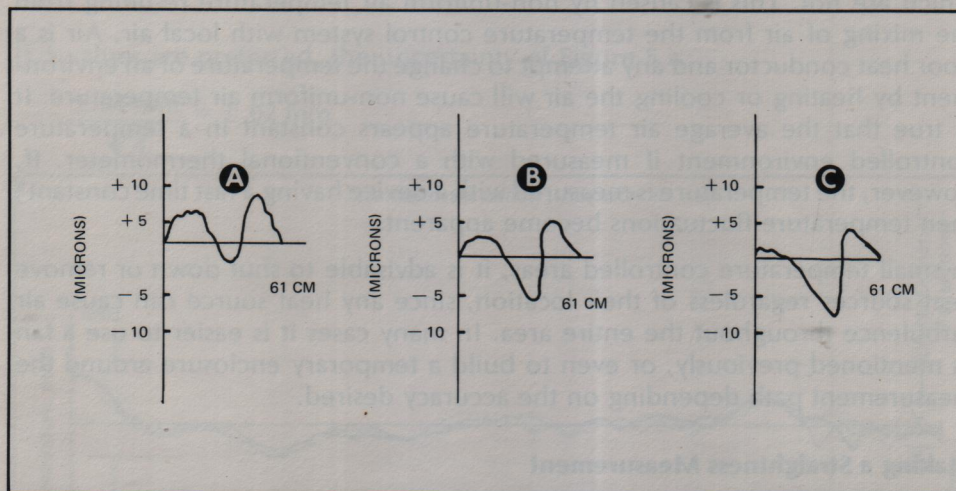


**Figure 3.** Straightness interferometer accuracy is a function of the flatness of the plane mirrors, which are small optical flats. These optical flats — not the laser beam — are the ultimate straightness reference for the system. The system can tolerate a modest flatness error ( $\delta$ ) of about a microinch over each two-inch mirror before the straightness error exceeds five microinches per foot of travel.



A second systematic source of error can be caused by improper fixturing of the straightness reflector. It should be noted that the straightness reflector represents a straightedge which is supported at only one end. If it is not rigidly supported it may tilt or rotate during a measurement. This will cause a change in slope of the data, and it is evidenced by poor repeatability at that point of travel where the interferometer is farthest from the reflector. This can be easily tested for by leaving the straightness interferometer at the far end of travel (farthest from the reflector), and observing the displayed readings for several minutes. If the displayed readings show a systematic drift during the time required to make a straightness calibration then the reflector should be fixtured more rigidly to the machine.

A similar drift can be caused by bending of the machine under test; particularly if it is being operated in a poor environment. This is caused by temperature gradients within the machine and must be considered as a machine error. The only solution to this is to test the machine in a better environment. Machines usually bend so slowly that this effect would not be obvious unless straightness measurements were made over an extended period of time, say several hours. Bending is typically evidenced by non-repeatability of successive runs, as shown in Figure 4. The data for Figure 4 was obtained by plotting the straightness of a steel straightedge which was slowly heated on one side.



**Figure 4.** Bending of the straightedge is evidenced by the changing straightness graphs A-C as the straightedge heats up.

### Random Errors

The most important factor affecting the accuracy and repeatability of the HP Straightness Interferometer System is air turbulence. Air turbulence, or inhomogeneity, is caused by random variations of the air temperature in the path of the two interfering beams. This, in turn, causes a random fluctuation in the interferometer reading which increases with increasing separation between the interferometer and reflector. The expected uncertainty in straightness due to air turbulence is about one micron for every metre of travel (1 ppm). It should be emphasized, however, that air turbulence is a direct result of air temperature differences within the measurement and can, therefore, be more or less than one micron per metre of travel. If air turbulence effects are significantly greater than this, then some effort should be made, if possible, to improve the environment.



# Improving the Environment

## Machine Shops

Although air is a poor heat conductor, it will, if left alone, come to thermal equilibrium and air turbulence effects will be minimized. Air turbulence is usually caused by heat sources in the vicinity of the measurement. If possible, such heat sources should be removed or shut down temporarily while straightness measurements are made. In most machine shops, thermal conditions will be best during an off shift when there is less activity in the area. Hence, the best time to make a straightness calibration would be during an off shift and many companies do this as a matter of practice.

If it is impractical to remove heat sources from the environment, air turbulence can be reduced by blowing air through the space between the interferometer and reflector by means of a fan. Care should be taken to ensure that hot air caused by heat sources is directed **away** from the measurement.

## Temperature Controlled Environments

Temperature controlled areas often exhibit greater air turbulence than areas which are not. This is caused by non-uniform air temperature resulting from the mixing of air from the temperature control system with local air. Air is a poor heat conductor and any attempt to change the temperature of an environment by heating or cooling the air will cause non-uniform air temperature. It is true that the average air temperature appears constant in a temperature controlled environment if measured with a conventional thermometer. If, however, the temperature is measured with a device having a fast time constant\*, then temperature fluctuations become apparent.

In small temperature controlled areas, it is advisable to shut down or remove heat sources regardless of their location, since any heat source can cause air turbulence throughout the entire area. In many cases it is easier to use a fan as mentioned previously, or even to build a temporary enclosure around the measurement path depending on the accuracy desired.

## Making a Straightness Measurement

When a straightness measurement using the HP Straightness Interferometer System is being performed it is important to record as many data points as possible during each straightness run. This can best be accomplished by taking measurements "on the fly" using either the timed print mode\*\*available on the HP 5505A Laser Display or externally sampling the Laser Measurement System.

Each run should be taken with the 5505A Laser Display in the SMOOTH or X10 mode, and a least squares, best fit slope calculation should be made. (A description of a least squares, best fit slope calculation is provided in the appendix.) To isolate effects of air turbulence, 50 to 100 data points should be taken per run.

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\* 0.1 second or less.

\*\*To determine the Print Rate (labeled "PRINTS-PLOTS/MIN") on the HP 5505A Display, use the following relation:

$$\text{PRINT RATE} = \frac{\text{Number of Data Points} \times \text{Feed Rate}}{\text{Distance Traveled}}$$



Air turbulence effects are indicated by random non-repeatability of successive runs and can easily be seen by plotting several successive runs (using the least squares, best fit calculation) on the same graph. To illustrate this measurement technique a long range straightness measurement was performed on a 28 metre (90 foot) machine tool with the results shown in Figures 5 through 9. In Figure 5, air turbulence caused non-repeatability of about .08 mm (.003 inches), whereas the slide straightness was about .280 mm (.011 inches).

Once the operator has plotted successive runs he must decide whether or not the non-repeatability is objectionable. If the non-repeatability is not objectionable (small compared to slide straightness or slide tolerance), the **average** of the runs plotted can be taken as an indication of slide straightness. The average of the four runs shown in Figure 5 is shown in Figure 6. The uncertainty of the average can be obtained by the following relation:

$$\text{Uncertainty of Average} = \frac{\text{Non-Repeatability of Runs Averaged}}{\sqrt{\text{Number of Runs Averaged}}}$$

In Figure 5, the uncertainty of average is:

$$\frac{.08 \text{ mm}}{\sqrt{4}} = .04 \text{ mm}$$

If  $\pm$  values are preferred, the uncertainty of Figure 5 is:

$$\frac{\pm .04 \text{ mm}}{\sqrt{4}} = \pm .02 \text{ mm}$$

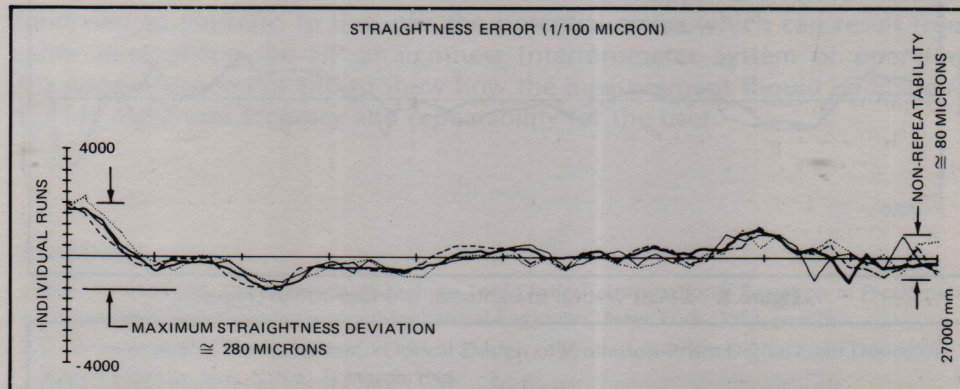


Figure 5. X-Axis horizontal straightness, four runs.

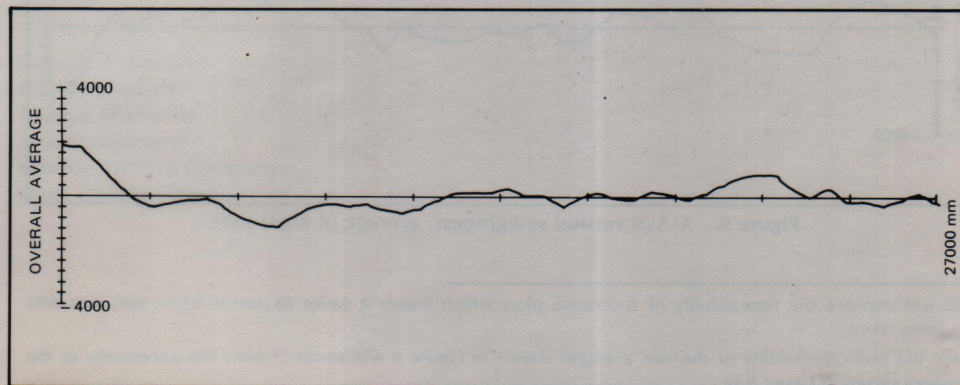


Figure 6. X-Axis horizontal straightness, average of four runs.



This uncertainty of average is small compared to the .280 mm overall slide straightness and therefore the average plot in Figure 6 is accepted as the slide straightness.

If non-repeatability of successive runs is objectionable (as shown in Figure 7), then the user will need to average more runs. In this case, it is better to plot successive **run averages** than to plot successive runs.\* Two successive four-run averages are shown plotted in Figure 8. At this point, it was decided that the non-repeatability was no longer objectionable and the two four-run averages were averaged yielding the plot shown in Figure 9. This was taken as an indication of slide straightness.

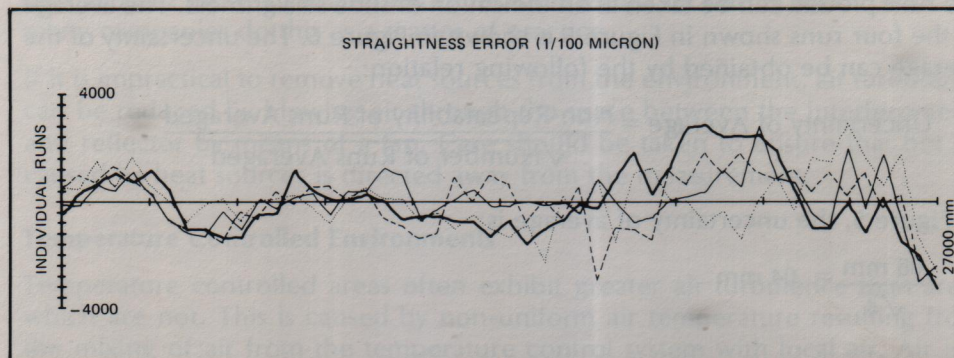


Figure 7. X-Axis vertical straightness, four runs.

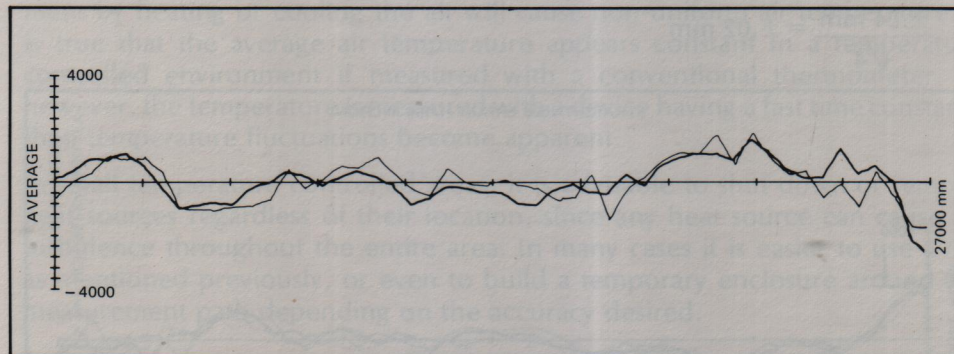


Figure 8. X-Axis vertical straightness, two four-run averages.

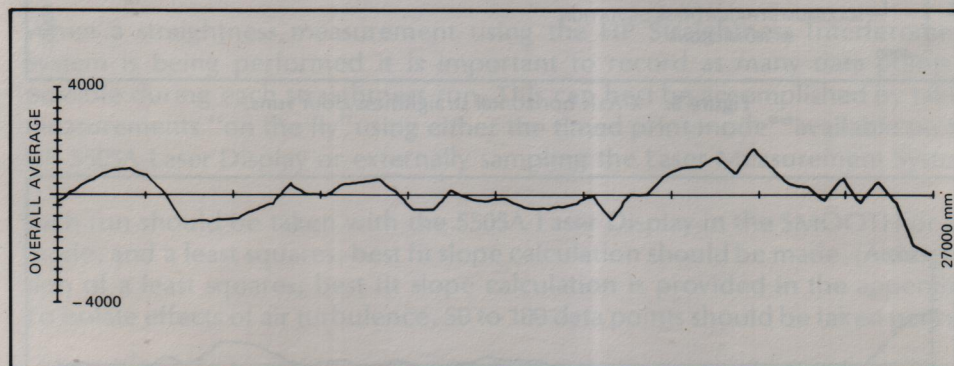


Figure 9. X-Axis vertical straightness, average of eight runs.

\*This will improve the repeatability of successive plots which makes it easier to decide when sufficient data has been taken.

Since the non-repeatability of the two averages shown in Figure 8 was about .1 mm, the uncertainty of the averages shown in Figure 9 is:

$$\frac{.10 \text{ mm}}{\sqrt{2}} = 0.07 \text{ mm} \quad \text{or} \quad \frac{.05 \text{ mm}}{\sqrt{2}} = \pm .035 \text{ mm}$$

Slide straightness as shown in Figure 9 is about .36 mm.



## Squareness and Parallelism Measurements

Air turbulence effects do not cause any significant error in squareness and parallelism measurements provided a least squares, best fit is used when processing the data. In this case, squareness and parallelism are given as an angular deviation in arc seconds between two straight-line, least squares, best fit approximations to the axes measured. Due to their random nature, air turbulence effects will "average out" during each run and will have little effect on the least squares, best fit approximation provided:

1. The maximum possible travel is measured on each axis, and
2. A sufficient number of data points is recorded on each axis.

It is not necessary to make multiple runs.

## Conclusion

The Hewlett-Packard Straightness Interferometer System has given the marketplace a technique for making very accurate, long distance, straightness measurements. However, since it is an optical measurement technique, it is subject, as are all optical measurements, to various fixturing and environmental errors which must be taken into consideration. This application note has been generated to illustrate to the user the potential errors which can result from either misapplying the HP Straightness Interferometer System or operating in a poor environment and to show how the measurement should be made to provide maximum accuracy and repeatability for the user.

## REFERENCES

1. See, for example, Kurt Rantsch, "Machine Tool Optics", International Research in Production Engineering, American Society of Mechanical Engineers. New York, 1963, p. 629.
2. R. A. Soref and D. H. McMahon, "Optical Design of Wollaston-Prism Digital Light Deflectors", Applied Optics, Vol. 5, No. 3, March 1966.
3. R. R. Baldwin, B. E. Grote, D. A. Harland, "A Laser Interferometer That Measures Straightness of Travel", Hewlett-Packard Journal, January 1974.

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## APPENDIX

### Least-Squares, Best Line Fit Calculation

#### Straightness

If the bisector of the Straightness Reflector is not exactly aligned with the axis of travel of the machine the Laser Display will appear to be incrementing in the positive or negative direction. A graphical view of this is shown in Figure A.

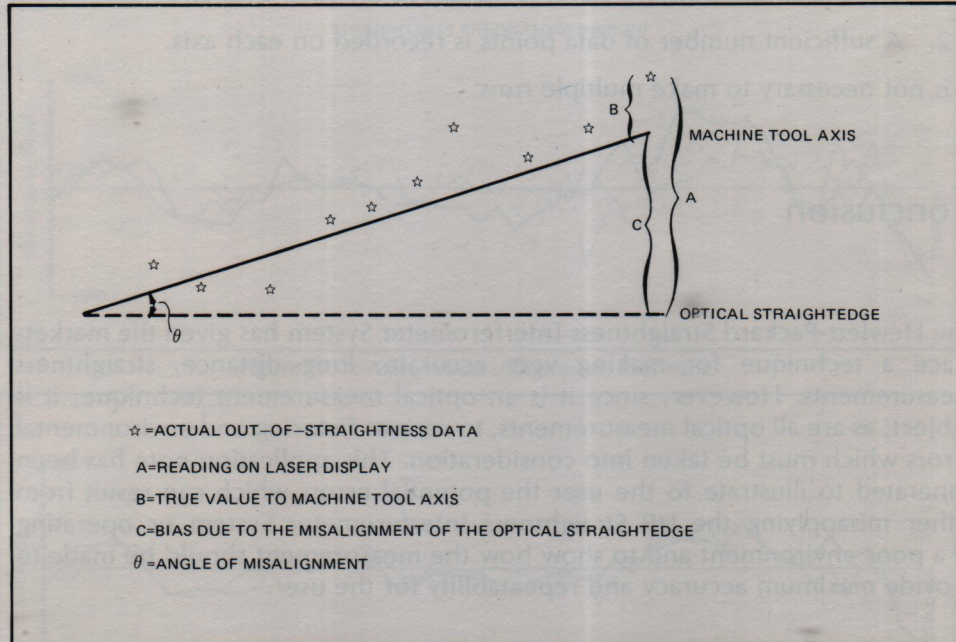


Figure A

One mathematical technique which the operator can use to reduce the value of  $\theta$  to zero is to perform a least-squares, best line fit on the data. This technique is illustrated at the end of this discussion.

#### Squareness

When one is making a single-axis straightness measurement there is no need to worry about the sign convention used other than to relate a positive deviation to the direction of the machine tool. However, when a squareness measurement is to be made the sign convention becomes critical in order to assure oneself of obtaining the true sign and magnitude of the squareness error.



Figure B shows the technique used to obtain a value of squareness.

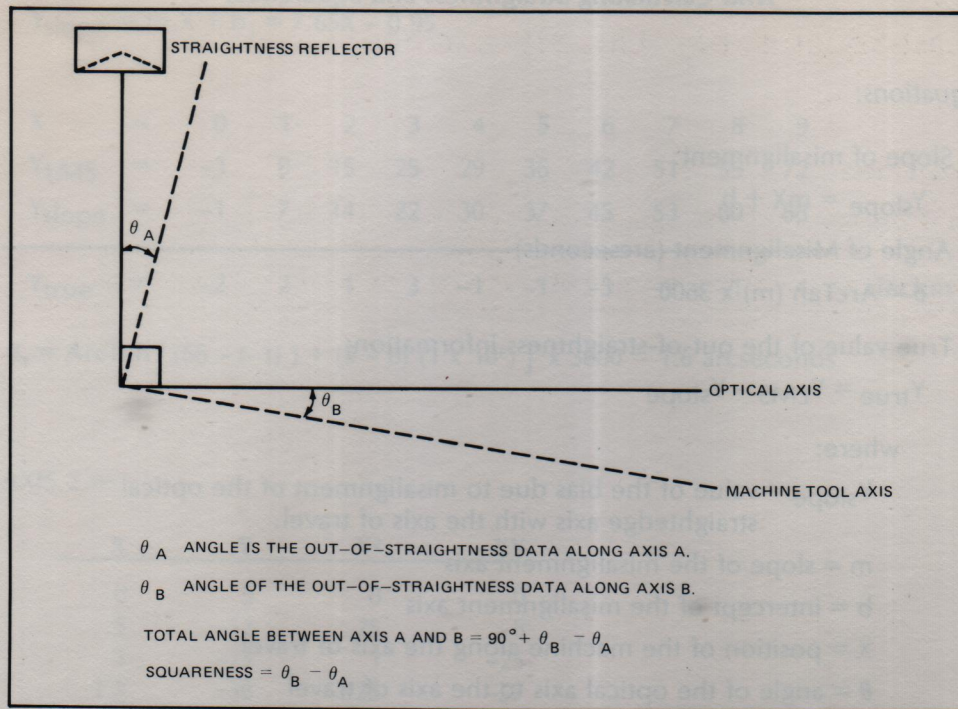


Figure B

If  $\theta_A$  and/or  $\theta_B$  has the wrong sign the final squareness error will be incorrect.

**Raw Data** (X10 Mode, Cal. No. = 1, Prism Error +2 arcseconds)

AXIS 1 —

Y<sub>LMS</sub> = -3 9 15 25 29 36 42 51 59 72 (μinches)

X = 0 1 2 3 5 5 6 7 8 9 (inches)

Number of points = N = 10

AXIS 2 —

Y<sub>LMS</sub> = 5 -12 -25 -38 -40 -48 -59 -62 -74 -88 (μinches)

X = 0 .5 1 1.5 2 2.5 3 3.5 4 4.5 (inches)

Number of points = N = 10

### LEAST SQUARES FIT

Equations used:

$$\text{Slope } m = \frac{\sum Y \sum X - N \sum XY}{(\sum X)^2 - N \sum X^2}$$

$$\text{Intercept } b = \frac{\sum X \sum XY - Y \sum X^2}{(\sum X)^2 - N \sum X^2}$$



## Example of Performing a Least Squares Best Line Fit And Calculating Straightness and Squareness

Equations:

Slope of misalignment:

$$Y_{\text{slope}} = mX + b$$

Angle of Misalignment (arcseconds)

$$\theta = \text{ArcTan}(m) \times 3600$$

True value of the out-of-straightness information:

$$Y_{\text{true}} = Y_{\text{LMS}} - Y_{\text{slope}}$$

where:

$Y_{\text{slope}}$  = value of the bias due to misalignment of the optical straightedge axis with the axis of travel.

$m$  = slope of the misalignment axis

$b$  = intercept of the misalignment axis

$X$  = position of the machine along the axis of travel

$\theta$  = angle of the optical axis to the axis of travel

$Y_{\text{true}}$  = true value of the out-of-straightness of the machine

$Y_{\text{LMS}}$  = reading on the laser display

AXIS 1 —

X	Y	X <sup>2</sup>	XY
0	-3	0	0
1	9	1	9
2	15	4	30
3	25	9	75
4	29	16	116
5	36	25	180
6	42	36	252
7	51	49	357
8	59	64	472
9	72	81	648
45	335	285	2139

$$m_1 = \frac{(335)(45) - (10)(2139)}{(45)^2 - (10)(285)} = 7.65 \text{ } \mu\text{inches/inch}$$

$$b_1 = \frac{(45)(2139) - (335)(285)}{(45)^2 - (10)(285)} = -0.95 \text{ } \mu\text{inches}$$



therefore:

$$Y_{\text{slope}} = m_1 X + b_1 = 7.65X - 0.95$$

X	=	0	1	2	3	4	5	6	7	8	9
Y <sub>LMS</sub>	=	-3	9	15	25	29	36	42	51	59	72
Y <sub>slope</sub>	=	-1	7	14	22	30	37	45	53	60	68

Y <sub>true</sub>	=	-2	2	1	3	-1	-1	-3	-2	-1	4	μinches
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$$\theta_1 = \text{ArcTan} [(68 - (-1)) \div (9 - 0) (1 \times 10^6)] \times 3600 = 1.6 \text{ arcseconds}$$

AXIS 2 —

X	Y	X <sup>2</sup>	XY
0	5	0	0
.5	-12	.25	-6
1	-25	1	-25
1.5	-38	2.25	-57
2	-40	4	-80
2.5	-48	6.25	-120
3	-59	9	-177
3.5	-62	12.25	-217
4	-74	16	-296
4.5	-88	20.25	-296
22.5	-441	71.25	-1374

$$m_2 = \frac{(-441)(22.5) - (10)(-1374)}{(22.5)^2 - (10)(71.25)} = -18.51 \text{ μinches/inch}$$

$$b_2 = \frac{(22.5)(-1374) - (-441)(71.25)}{(22.5)^2 - (10)(71.25)} = -2.45 \text{ μinches}$$

therefore:

$$Y_{\text{slope}} = m_2 X + b_2 = -18.51X - 2.45$$

X	=	0	.5	1	1.5	2	2.5	3	3.5	4	4.5
Y <sub>LMS</sub>	=	5	-12	-25	-38	-40	-48	-59	-62	-74	-88
Y <sub>slope</sub>	=	-2	-12	-21	-30	-39	-49	-58	-67	-76	-86

Y <sub>true</sub>	=	7	0	-4	-8	-1	1	-1	5	2	-2	(μinches)
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$$\theta_2 = \text{ArcTan} [(-86 - (-2)) \div (4.5 - 0) (1 \times 10^6)] \times 3600 = -3.8 \text{ arcseconds}$$

$$\text{Squareness} = \theta_2 - \theta_1 + \text{Prism Error} = -3.8 - 1.6 + 2 = -3.4 \text{ arcseconds}$$